Learning processes in pre-service teacher training for secondary school mathematics teachers

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Abstract

In this paper, I present research in which I explore the learning processes of the future teachers who participated in a secondary mathematics methods course. I introduce the notion of didactic analysis as a conceptualisation of the activities that a teacher should perform in order to design, implement and assess didactic units. I specify the idea of didactic knowledge as the knowledge required to perform didactic analysis. Based on four related studies, I describe and characterise the future teachers' development of didactic knowledge by establishing four developmental stages of their didactic knowledge and characterising, from several view-points, the evolution of the meanings that the future teachers constructed throughout the course. The results of these studies reveal several aspects of the complexity involved in the training of future teachers of secondary school mathematics. They show the need to study further the design and development of this kind of program and to take into account the role that the trainers can play in the learning of future teachers.

Keywords: Initial Teacher Training, Mathematics, Secondary Education, Learning, Didactic knowledge, Didactic analysis

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Resumen

En este artículo presento una investigación en la que exploré el proceso de aprendizaje de los futuros profesores que participaron en una asignatura de formación inicial de profesores de matemáticas de secundaria. Introduzco el análisis didáctico como conceptualización de las actividades que un profesor debería realizar para diseñar, implementar y evaluar unidades didácticas y concreto la idea de conocimiento didáctico como el conocimiento necesario para realizar el análisis didáctico. Con base en cuatro estudios interrelacionados, describo y caracterizo el desarrollo del conocimiento didáctico de los futuros profesores, al establecer cuatro estados de desarrollo del conocimiento didáctico y caracterizar, desde diferentes perspectivas, la evolución de los significados que los futuros profesores construyeron a lo largo de la asignatura. Los resultados de los estudios ponen de manifiesto varios aspectos de la complejidad de la formación inicial de profesores de matemáticas de secundaria y destacan la necesidad de profundizar en el diseño y desarrollo de este tipo de planes de formación y en el papel que los formadores pueden jugar en el aprendizaje de los futuros profesores.

Palabras clave: Formación Inicial del Profesorado, Matemáticas, Educación Secundaria, Aprendizaje, Conocimientos Didáctica, Análisis Didáctico.

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Introduction

Teacher training has become one of the main research focuses in mathematics education in the last fifteen years (Sfard, Hashimoto, Knijnik, Robert and Skovsmose, 2004). Interest focuses on four central questions (Gómez, 2007, pp. 2-4):

1. What characterises the effective and efficient action of the teacher in the mathematics classroom?
2. What knowledge, capacities and attitudes should effective and efficient teachers have?
3. How should initial training programmes for secondary school mathematics teachers be designed and implemented so that they support and foster the development of such knowledge, capacities, and attitudes?
4. What characterises the learning processes of future secondary school mathematics teachers who participate in this kind of initial training programme?

The study that I describe in this article is framed by the sphere of action of these four questions.[1] For each question, I determine a specific work context. For the first question, I propose from a conceptual perspective, a description of the ideal procedure that the mathematics teacher should perform in designing, putting into practice and evaluating didactic units—the didactic analysis. Second, I establish the knowledge and abilities that the teacher should have and develop to perform the didactic analysis—the didactic knowledge. As to training plans, I focus attention on the process of curriculum design—the planning of didactic units. In addition, I circumscribe the work within the course *Mathematics Education in Secondary School* at the University of Granada. This means focusing attention on the initial training of secondary school mathematics teachers in the Spanish context. Finally, I study and characterise from an evolutionary and socio-cultural perspective the learning of the future teachers who took this course during the academic year 2000-2001.

A Functional View of the Initial Training of Secondary School Mathematics Teachers

At the University of Granada, we have tackled the problem of the design of training courses from a functional perspective. Instead of starting from what we believe the teacher should know, we ask ourselves what the teacher should be able to do. Thus, we first reflect on what activities the teacher can perform to promote student learning. From this reflection, we establish the competences that we hope the future teacher will develop in his or her training process.

We focus our attention on developing the competence of the teacher's planning. Planning is one of the most important activities in the teacher's work (Ball, 2003, p. 3; Van Der Valk and Broekman, 1999) and constitutes one of his or her competences (Kilpatrick, Swafford and Findell, 2001, p. 380; Niss, 2003; Recio, 2004; Rico, 2004). The didactic analysis, introduced by Rico (Rico, 1992, § III.2.1; Rico, 1997b, p. 55), which we have been developing recently (Gómez, 2002a, 2007) is a conceptualisation at the local level of planning. It is constituted at a new level of the curriculum (Gómez, 2002b, p. 256), tackles the problem of the gap between global and local curriculum design (Rico, 1997a; Segovia and Rico, 2001), is framed by a functional view of the mathematics curriculum (Rico, Castro, Castro, Coriat and Segovia, 1997b, p. 284), and is shaped by a set of notions about the teaching of mathematics—the curriculum organisers (Rico, 1997a, p. 44).

When planning is local, the focus of the teacher's attention is a specific mathematics topic. At this level, the teacher's planning should take into account the complexity of the mathematics content from various points of view: "when mathematics is taught from a pluralist perspective, one can see from multiple perspectives—perspectives that motivate teachers to consider not only the different meanings of mathematics but also the variety of these meanings in the teaching of mathematics" (Cooney, 2004, p. 511). In fact, negotiating and constructing this multiplicity of meanings should be one of the central goals of interaction in the classroom. This is the position that Rico and his collaborators have proposed since the early 1990s as an approach to the planning of didactic units in Spain (Rico, 1992; Rico, 1995, 1998a, 1998b, 1997c; Rico et al., 1997a). Their proposal centres on the idea that the planning of a didactic unit or of one hour of class should be based on the exploration and structuring of the different meanings of the mathematics structure to be planned. The "curriculum organisers" proposed by Rico that I will now illustrate are conceptual and methodological tools that enable the teacher to obtain, organise and select information on these multiple meanings. Thus, from another perspective of our functional view of the initial training of secondary school mathematics teachers, we view the curriculum organisers as analytic tools with a practical purpose.

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[1] I performed this study as part of my doctoral thesis at the University of Granada, under the direction of Luis Rico Romero.

Didactic Analysis: A Procedure for Organising the Teaching of Mathematics

In the specific context of planning an hour of class or a didactic unit, the teacher can organise the teaching based on four analyses (Gómez, 2002b, 2007):

1. *subject matter analysis,* as a procedure by which the teacher identifies and organises the multiplicity of meanings of a concept;
2. *cognitive analysis,* in which the teacher describes his hypotheses about how the students can progress in the construction of their knowledge of the mathematical structure when they face the tasks that will make up the teaching and learning activities;
3. *instruction analysis,* in which the teacher designs, analyses, and chooses the tasks that will constitute the teaching and learning activities to be taught; and
4. *performance analysis,* in which the teacher determines the capacities that the students have developed and the difficulties that they may have expressed up to that point.

I use *didactic analysis* to refer to a cyclical procedure that includes these four analyses, attends to the factors conditioning the context and identifies the activities that the teacher should ideally perform to organise the teaching of a specific mathematical content. The description of a cycle of didactic analysis follows the sequence described in Figure 1.

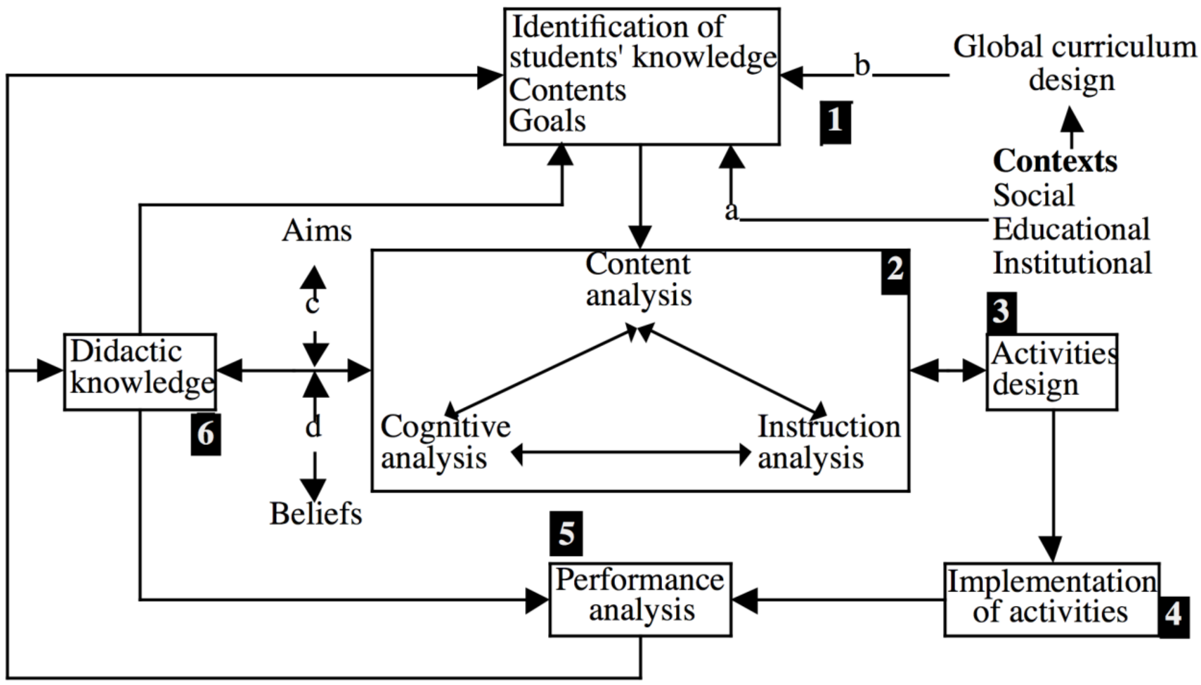

***Figure 1*. Cycle of didactic analysis**

The cycle of didactic analysis begins with the determination of the content to be treated and the learning goals to be achieved. It starts from the teacher's perception of the students' understanding based on the results of the performance analysis in the previous cycle and taking into account the social, educational and institutional contexts that frame the instruction (Box 1 of Figure 1). From this information, the teacher begins planning with subject matter analysis. The information that emerges from subject matter analysis serves as the basis for cognitive analysis by identifying and organising the multiple meanings of the concept to be taught. The cognitive analysis can then give rise to a revision of the subject matter analysis. This relation between the analyses is also established with instruction analysis. Its formulation depends on and should be compatible with the results of the subject matter analysis and the cognitive analysis, but performing it can simultaneously generate the need to correct earlier versions of these analyses (Box 2). In cognitive analysis, the teacher selects some reference meanings and, based on these and on the learning goals that have been imposed, identifies the capacities that he seeks to develop in the students. The teacher also formulates conjectures on the possible paths by which students can develop their learning when they tackle the tasks that make up the lesson. The teacher uses this information to design, evaluate and select these tasks. As a result, the choice of tasks that compose the activities should be consistent with the results of the three analyses, and the evaluation of these tasks in the light of the analyses can lead the teacher to perform a new cycle of analysis before choosing the definitive tasks that compose the teaching and learning activities (relation between Boxes 2 and 3). The teacher puts these activities into practice (Box 4) and, in doing so, analyses the students' actions to obtain information that serves as the starting point of a new cycle (Box 5). Didactic knowledge (Box 6) is the knowledge that the teacher brings into play during this process.

Subject matter analysis is shaped by three curriculum organisers —conceptual structure, systems of representation, and phenomenology— that enable us to tackle systematically the meanings of a concept for school mathematics (2007, pp. 36-55). Next, I will use an example to present *some* aspects of the meaning and the uses of the curriculum organiser conceptual structure (and its relationship to the curriculum organiser systems of representation) in order to make us aware of the complexity of the subject matter analysis in particular and of the didactic analysis in general.

I use the expression *conceptual structure* to refer to three aspects of every concept of the school mathematics content:

1. *Mathematics structures involved*. Every mathematics concept is related to at least two mathematical structures: (a) the mathematical structure that the concept configures and (b) the mathematical structures of which it forms part. For example, the concept of the quadratic function configures a mathematical structure in which structural relations are established between concepts such as quadratic equation, parameter, focus and vertex (see Figure 2). In addition, the concept of quadratic function forms part, for example, of the mathematical structure corresponding to the concept of function.
2. *Conceptual relations*. I stress the relations that are established between the concept and (a) the concepts of the mathematical structure that this concept configures (e.g., the relation between the quadratic function and the quadratic equation), (b) the objects that are specific cases of this concept (that is, the objects that saturate the predicate; e.g., $f(x) = 3x^2 - 4$ as a specific case of quadratic functions of the form $f(x) = ax^2 + c$), and (c) the concepts that belong to the mathematical structure of which the concept forms part (e.g., the relation between the quadratic function and continuous functions).
3. *Relations of representations.* Exploring the meanings of a concept requires systems of representation, since these enable us to identify the ways in which the concept appears. On taking into account the systems of representation, we can point out several relations (see Figure 2): (a) the relation between two signs that designate the same object or concept, within the same system of representation (invariant syntactical transformations —e.g., as a result of completing squares), (b) the relation between two signs that designate the same object or concept belonging to different systems of representation (translation between systems of representation —e.g., the relation between parameters of a symbolic form and elements of the graphic representation and (c) the relation between two signs that designate two different objects or concepts within the same system of representation (variant syntactical transformations —e.g., as a result of applying a graphics transformation).

When we explore the meanings of a concept in school mathematics, we should thus take into account three kinds of "elements" and two groups of relations between these elements.

The elements are:

- the *objects,* as specific cases of a concept, which configure its extension,
- the *concepts*, as predicates that are saturated by the objects and, in turn, form mathematical structures, and
- the *mathematical structures*, which are shaped by concepts.

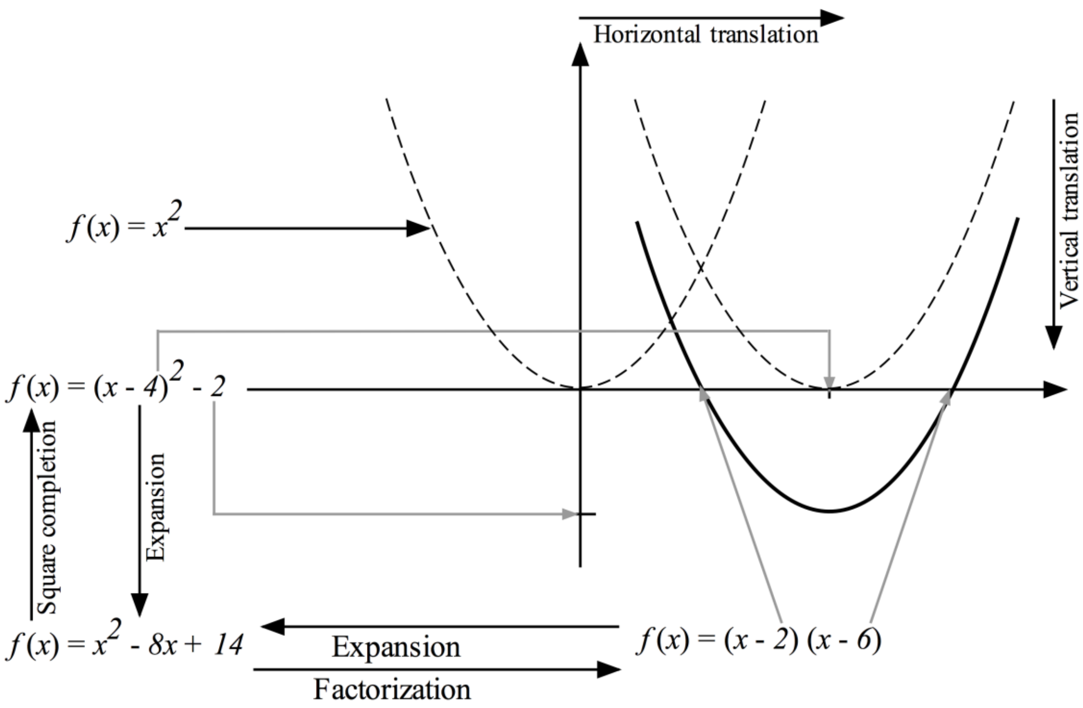

Figure 2. ***Operations in systems of representation***

On the other hand, the relations described in Points 2 and 3 above can be grouped into two categories, which I call *vertical relations* and *horizontal relations.* Vertical relations refer to relations between the three kinds of elements: Object → Concept → Mathematical structure. Horizontal relations refer to the relations between signs in their different systems of representation (relations between representations).

Tackling the meanings of a concept from the perspective of a conceptual structure involves identifying and organising the elements (objects, concepts and mathematical structures) and the relations (horizontal and vertical) corresponding to this concept. In Gómez (2007), I describe my proposals for other facets of the didactic analysis.

Didactic Knowledge

I will use the expression *didactic knowledge* to refer to the knowledge and skills that are necessary to perform a didactic analysis of a mathematical topic. In the literature on mathematics education, we find a variety of possible meanings for the curriculum organisers that are brought into play in didactic analysis. I identify this knowledge as *disciplinary didactic reference knowledge*.

For the purpose of designing a course for initial training, it is necessary to interpret the disciplinary didactic reference knowledge and select some particular meanings for each of the curriculum organisers. This is the *didactic reference knowledge for the course*, that is, the combination of knowledge and skills that the designers of this training plan take as an option within the disciplinary didactic reference knowledge and that they hope that future teachers will interpret and construct as one of the results of their training.

I consider three aspects of each curriculum organiser: its meaning, its technical use and its practical use. The meaning of a curriculum organiser refers to the option that the trainers take to be the meaning of the curriculum organiser within the range of possible meanings that exist in the literature on Mathematics Education. This meaning supports a combination of ideal strategies of analysis of a mathematical concept that shape the *technical use* of each curriculum organiser. The technical use stresses the character of analytic tool involved in each curriculum organiser. The analysis of a mathematical structure by means of each curriculum organiser has a practical purpose: the information that emerges from these analyses should ground the planning that we hope the groups of future teachers will perform. I call the *practical use* of the curriculum organiser the combination of strategies and techniques needed to use the information that emerges from the analysis of the mathematical structure with this curriculum organiser in the rest of the analyses that shape the didactic analysis and in the design of the didactic unit. In the previous section, I presented some aspects of the meaning and the technical use of the conceptual structure. The information that emerges from this technical analysis of the concept (the quadratic function) can be used, for example, to identify the students' errors and difficulties or to specify the subject matter of the didactic unit to be planned. These would be practical uses of this curriculum organiser.

When participating in an initial training course, future teachers (and groups of future teachers) interpret the didactic reference knowledge and construct knowledge (individual or collective). This is the *didactic knowledge of the future teacher or group of future teachers*. Such knowledge is in permanent evolution. My empirical interest in this research project focuses on describing, characterising and explaining (in part) the processes by which groups of future teachers develop their didactic knowledge. I will thus refer to the meaning that a future teacher or group of future teachers has (or develops) for a curriculum organiser.

Future Teachers' Learning

The initial training of mathematics teachers is a complex social practice. The socio-cultural approach attends to this complexity (Adler, 1998; Lerman, 2001, p. 45). Research on the training of teachers from this perspective enables us to explore and characterise aspects of the teachers' process of change that traditional psychological perspectives do not allow us to see (Stein and Brown, 1997, p. 155), as the psychological approaches tend to study the development process of individual teachers in highly structured contexts.

Social Theory of Learning

After taking into account the previous arguments and the goals of this research, I chose Wenger's social theory of learning (1998) as the conceptual foundation for the learning of future teachers. This theory views learning as a social phenomenon that forms part of the experience of participating socially in the world. The idea of participation refers to "a more encompassing process of being an active participant in the *practices* of social communities and constructing *identities* in relation to these communities" (p. 4, italics in the original). Learning as social participation is based on four notions:

- *meaning*, as our changing ability (individual and collective) to experience our life and the world as meaningful;
- *practice*, as our resources, plans and shared historical and social perspectives that can support the mutual commitment to action;
- *community*, as the social configurations in which our undertakings are defined as worthwhile and our participation is recognised as competent;
- *identity*, as expression of how learning changes who we are and the creation of personal histories of becoming in the context of our communities.

The notion of community is based on three notions: *mutual commitment*, as the commitment to actions whose meaning is negotiated and that generate relationships between people; a *joint enterprise*, which is negotiated collectively and continually and which generates mutual responsibility and determines what is valued, discussed and shown; and a *shared repertoire*, which includes the resources for negotiation of meaning, the discourse that enables us to make meaningful statements about the world and the styles for expressing forms of membership and identity as members.

In the context of the course in which I performed the research, I considered that the information contained in the transparencies and the classroom performance of the group's members constituted expressions of the meanings that this group had constructed so far. I use the term *partial meanings* to refer to these meanings. As mentioned in the previous section, I am especially interested in the evolution of the meanings of the future teachers with respect to the curriculum organisers of the subject matter analysis. I call them "partial" because I wish to emphasise, as Wenger suggests, that the meanings that one group has constructed at a given moment in its training are always open to improvement. They are the result of what the group has learned up to that point, as a result of a continuous and dynamic process of negotiating meanings. In other words, at any given moment during the course, each group has achieved a certain development of its shared repertoire, and its productions (transparencies and performance) are expressions of this shared repertoire.

Theory of Instrumental Genesis

The relation between the activities the future teacher is expected to perform, the meanings of the curriculum organisers in didactic analysis and the kinds of knowledge involved show the complexity of didactic knowledge and of the initial training of secondary school mathematics teachers. Didactic knowledge, as the knowledge brought into play and developed in performing didactic analysis, is *knowledge for action*. Developing this knowledge requires future teachers to be able to transform the curriculum organisers that make up the didactic analysis into instruments. The development of the didactic knowledge of future teachers is based on an interplay between theory and practice that can be characterised by adapting the theory of instrumental genesis (Rabardel, 2003; Rabardel and Bourmaud, 2003; Vérillon, 2000): This is achieved by using the curriculum organiser (the instrument) as a mediator between future teachers and the concept on which they are working, a mediator that they construct and about which they develop meanings concerning both the notion and the concept. The idea of instrumental genesis arises from the argument that an artefact becomes an instrument to the extent that three processes take place:

1. *instrumentalisation*, as the process in which the subject transforms and adapts the artefact to his needs and circumstances (Rabardel and Bourmaud, 2003, p. 673).
2. *instrumentation*, as the process in which action plans are generated (p. 673), that is, abilities to apply the tool in order to perform meaningful tasks (Kaptelinin, 2003, p. 834) that, in turn, are transformed into techniques (Artigue, 2002, p. 250). A technique is a mix of reasoning and routine procedures that enable the completion of a task (p. 248).

3. *orchestrated integration*, by which the tool is integrated with other artefacts (Kaptelinin, 2003, p. 834).

When performing these tasks, the groups of future teachers develop processes of instrumentalisation, instrumentation and orchestrated integration. That is, they transform and adapt the meaning that they assign to the curriculum organiser (instrument), develop plans for applying the tool—either to obtain information about the meanings of the concept (object) or to use this information in other analyses—and integrate the use of a specific instrument (e.g., the systems of representation) into other instruments for the design of the didactic unit. *It is through using the instrument (curriculum organiser) as mediator among the group of future teachers (subject) and the concept on which they are working (object) that the group constructs and develops meanings about both the curriculum organiser and the concept*. This activity, which involves the generation of techniques, transforms the group's practice.

Instrumental genesis takes place in this process of performing tasks: the artefact (the curriculum organiser) is transformed into an instrument to the extent that the group of future teachers develops plans to complete the tasks with the help of the instrument. And it is in this process of instrumental genesis that the group negotiates the meanings (of the curriculum organiser, of the object and of the plans) that are brought into play in the activity, reified in the shared repertoire and manifested in their productions and performance in the classroom. As a result, the notion of instrumental genesis allows me —for the specific context of this research project— to specify and conceptualise the general process of negotiation of meaning proposed by Wenger into a more specific process that characterises the activities that the groups of future teachers perform outside class.

Inspired by the notion of "information quality" (Miller, 1996; Pipino and Wang, 2002; Strong, Lee and Wang, 1997; Wand and Wang, 1996) currently being developed in the discipline of management of organisations, I reformulate and organise the attributes of the quality of the information contained in the transparencies of groups of future teachers and expressed in their class presentations in three dimensions, which I call *development factors*: variety, organisation and role. The factor *variety* attempts to include the idea that, for each curriculum organiser of the subject matter analysis, the description of a mathematical structure can be made with a larger or smaller quantity of information, depth or complexity. The factor *organisation* indicates how, within a production, the information gathered for one or more curriculum organiser of the subject matter analysis is organised. Finally, the third organising factor of the attributes of a production is the putting into practice of the information gathered for a given curriculum organiser. I call this factor *role*, because it seeks to reflect the role that each curriculum organiser of the subject matter analysis plays in other aspects of the didactic analysis.

Context and Design of the Research

In the previous sections, I specified the concepts and theories that enabled me to give meaning to the expression "characterise the learning processes of the future secondary school mathematics teachers". I can now formulate some specific research questions:

1. What are the partial meanings, with respect to the notions of subject matter analysis that emerge in the development of didactic knowledge when groups of future teachers participate in the course?
2. How can we describe the evolution of these partial meanings in terms of states and factors of development?
3. How can we characterise the states of development, if they can be determined?
4. Is it possible to explain these states of development, and the associated partial meanings, in terms of what happens in the community of learning in the classroom and in the community of learning in one of the groups?

In the academic year 2000-2001, 36 future teachers registered for the course, 25 women and 11 men. All were students pursuing a Bachelor's Degree in Mathematics at the University of Granada. They were in their fourth or fifth year of the Specialisation in Methodology. During the first weeks of the course, the future teachers were organised into eight groups: five groups of five, two groups of four and one group of three members. These groups remained stable throughout the course. At the beginning of the second trimester, each group chose a mathematics topic on which to develop its didactic analysis and produce the design of a didactic unit. The topics chosen were the following: graphs and functions, series, decimal numbers, probability, conic sections, the sphere, quadratic function and systems of linear equations. The syllabus was followed strictly, with delays of one class period only. An outline of modules was implemented, according to which the curriculum organisers were developed in similar ways (see Figure 3).

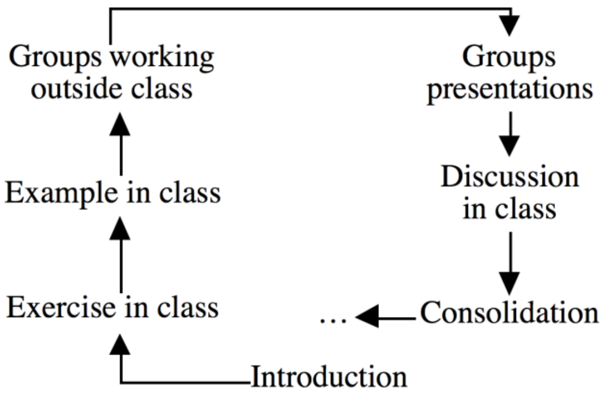

Figure 3. ***Didactic analysis methodological cycle***

This work plan gave rise to three kinds of information that I used in the empirical studies whose results I will present in the next section:

1. the information contained in the transparencies used by the groups of future teachers and by the trainers to give their class presentations,
2. the information contained in the transcriptions of the audio recordings of the class sessions and
3. the information contained in the final projects presented by the groups of future teachers.

I used two additional information sources:

4. the transcriptions of the audio recordings of semi-structured interviews with the groups on conic sections and arithmetic and geometric progressions as they were finishing the didactic analysis and at the end of the course and
5. the transcriptions of the audio recordings of the work sessions outside the classroom by the group on quadratic function in the process of developing its presentations and the final project.

Development of Didactic Knowledge

I organised the experimental dimension of the research into four interrelated studies, whose results I will now summarise.

Four States of Development

In the first study, I sought to identify the most representative attributes in the transparencies produced by the groups of future teachers, to define some of the variables to be analysed from these attributes, to verify that these variables followed stable patterns over time, and to identify and characterise the development of didactic knowledge from these states of development. To achieve this, I designed and put into practice a process of analysis and codification of the transparencies to establish 12 variables for analysis, in which each transparency was an observation in terms of these variables. I then developed a methodological procedure, discrepancy analysis, to classify the observations. This procedure enabled me to establish the four states of development that represent the best fit with the observations.

The schema for codifying and analysing the information with which I obtained the results is based on a cyclical process that seeks to minimise discrepancies. The states of development that emerge from this process identify the combinations of values (or ranges of values) of the variables to which, as a whole, the observations for a given task are best adapted. These combinations of values of variables can thus be considered representative of the most significant states of development of didactic knowledge in the groups of future teachers. This procedure enabled me to characterise these four states in terms of the curriculum organisers of the subject matter analysis and the development factors, as follows:

State 1 is a basic state in which the conceptual structure lacks complexity, various criteria of organization are used without consistency, at most one system of representation is used (without connections), and there is no variety in the phenomenological analysis. Only three groups have observations classified as belonging to this state. This suggests that is it a state that can be surpassed with the prior knowledge and didactic intuitions that the future teachers bring initially to the task.

State 2 is a transition state. There is some complexity in the conceptual structure, and variety begins to appear in the systems of representation, although there is still no variety in the phenomenological analysis.

State 3 shows an advance in all of the variables except those of role and consistency. The conceptual structure is complex, with an intermediate level of organisation. There is variety in the systems of representation and the number of connections. There is some variety in the phenomenological analysis.

State 4 achieves full complexity in the phenomenological analysis, and we can see the consistent use of the information for the completion of tasks.

Table 1 presents the final classification of the nine productions of the eight groups into each of the four states. Each row represents a group of future teachers and their corresponding observations, organised chronologically. Thus, for example, the observations corresponding to Group 7 were assigned successively to the following states: 2, 2, 3, 3, 3, 3, 4, 3 and 4.

Table 1. ***Final assignment of observations to states***

	Observation								
Group	1	2	3	4	5	6	7	8	9
1	2	2	2	3	3	4	3	4	4
2	1	2	3	3	3	3	3	3	4
3	1	2	2	3	3	3	3	3	3
4	2	2	2	2	2	3	3	3	3
5	2	2	3	3	3	3	3	4	4
6	2	2	2	3	3	3	2	3	4
7	2	2	3	3	3	3	4	4	4
8	1	2	2	2	3	3	3	2	2

The characterisation of the states and classification of the observations according to them is the main result of this study. This result confirms my initial conjecture: that the didactic knowledge of the groups of future teachers evolved according to stable patterns. The groups progressed in the development of their didactic knowledge at different rhythms of progress and levels of advancement. Analysis of the discrepancies in each variable sheds light on which notions presented more difficulty. For example, the notion of connection presented a high number of discrepancies with positive difference: In spite of the repeated efforts in instruction, the productions of the groups of future teachers had a level of connection lower than that expected. Something similar occurred, although to a lesser degree, with the notions of variety of phenomena, variety in systems of representation, complexity and systems of representation as an organiser of the conceptual structure.

The partial meanings of the groups of future teachers for the curriculum organisers of the subject matter analysis underwent diverse transformations and were consolidated to the extent that subsequent tasks led the groups of future teachers to bring their knowledge of these notions into play to solve other problems (for example, to identify errors or design an evaluation activity). In terms of the theory of instrumental genesis, the artefact (the curriculum organiser) was transformed into an instrument to the extent that the groups of future teachers developed plans for completing the tasks with the help of the instrument. The process of instrumental genesis took time: it required the groups of future teachers to negotiate meanings (of the curriculum organisers, of their mathematical topic and of their techniques) and to reify these partial meanings (in different forms) on subsequent occasions when the groups presented their productions in class. This process explains some of the differences between the observations and the pattern expected from the classification of the four states.

Complexity of Didactic Knowledge

In the second study, taking into account the specificity of the information with respect to the topic of each group, I sought to identify and characterise the partial meanings that the groups of future teachers showed with respect to the ideas of the subject matter analysis, to describe the evolution of these meanings throughout the course, and to deepen understanding of the characterisation of the states of development of didactic knowledge. To perform this study, I used three sources of information: (a) the information provided by the groups of future teachers in their transparencies; (b) the transcriptions of the audio recording of the interaction that took place during the class sessions; and (c) the transcriptions of the audio recording of interviews with two groups of future teachers (conic sections and arithmetic and geometric progressions) at the end of the sessions on subject matter analysis and at the end of the course. I performed a cyclical exploratory process in which I codified and analysed the information available from the three previously-mentioned sources. The process was based on the simultaneous analysis of the transparencies of the groups of future teachers and of the transcriptions of the recordings of class discussion and the interviews with the two groups. I was thus able to characterise the partial meanings and their evolution with respect to conceptual structure, systems of representation and phenomenology.

I established the evolution in the organisation of the conceptual structure: it moves from a list to an organisation by systems of representation. One can also see, however, a formal approach to the organisation of the conceptual structure. This formal approach did not enable its organisation by the systems of representation: the lower the number of criteria, the greater the organisation and complexity.

The future teachers established a hierarchy of systems of representation that became evident in the variety, organisation and putting into practice of this curriculum organizer. They tended to see the symbolic representation as the conceptual view of the concept and the graphic representation as complementary and equivalent to the notion of representation. They had difficulty with the notion of connection.

Phenomenology was the notion that gave the groups of future teachers the most difficulties They used many approaches and criteria of organisation for the phenomenological analysis but did not manage to develop a global vision of the procedure. Although they recognised evolution in their didactic knowledge of the notion, the future teachers did not put into practice the information that they produced in analysing the mathematical topics from the phenomenological perspective.

When each group chose its topic, its members assumed that the topic was simple, mathematically speaking. This view changed as they developed their topic in greater depth. The groups of future teachers expanded their view of what a mathematical structure was. Their experience as mathematics students and as teachers in private classes had surely reinforced an essentially formal view of mathematical concepts. It is possible that this way of seeing things was at the heart of the difficulties they experienced in appreciating the complexity behind each topic. However, as instrumental genesis took place and the groups of future teachers progressed in the identification and organisation of the different meanings of the mathematical concept, they became aware of its complexity. The results of this study show that most of the groups of future teachers were able to tackle this complexity from conceptual and representational perspectives. However, to some extent, this complexity overwhelmed them when they had to use the results of their analyses for didactic purposes. When it was expected that they would use the information gathered to design tasks or assessment activities, the groups of future teachers reverted to the traditional elements: a conceptual view that uses basic systems of representation (symbolic, graphic and numerical) and does not take advantage of the phenomenological analysis.

The analysis shows that the groups of future teachers negotiated and constructed the meaning of the curriculum organisers to the extent that they tried to use it in practice on a specific topic. Advances were achieved when, having proposed a solution to the problem, the groups of future teachers compared their solution to the solutions of the other groups and contrasted their position with the opinions, comments and critiques of their classmates and trainers. In this process, the future teachers were able to recognise the deficiencies in their initial solution, take into account the critiques received on it, research the scholarly literature and discuss new proposals to arrive at a new solution that arose from agreement among the members of the group.

Putting Didactic Knowledge into Practice

In this third study, my purpose was to explore the putting into practice of the information gathered for the curriculum organisers of the subject matter analysis. To do this, I analysed the final projects presented by the groups of future teachers and established what information of that proposed in the subject matter analysis was used in the analyses of the didactic analysis and in the design of the didactic unit. I also examined whether any information corresponding to the subject matter analysis was used in the other analyses and in the design of the didactic unit but was not registered explicitly in the section of the document on subject matter analysis.

The analysis showed a weak relation between the information gathered for the curriculum organisers of the subject matter analysis and their use in the other analyses and in the design of the didactic unit. The groups of future teachers used the information that emerged from the subject matter analysis only partially. They did not necessarily succeed in developing a global, integrated vision of the subject matter analysis in particular and of the didactic analysis in general as a tool for the design of didactic units.

A Community of Practice

Given that the analysis of the transparencies and presentations does not provide information on the process by which each group of future teachers negotiates meanings and advances in its learning process, the fourth study proposed to characterise the learning processes of the groups of future teachers. To do this, I analysed the transcription of the recordings of the work done outside class of one group of future teachers. This process of codification and analysis was based on an adaptation and operationalisation of the social theory of learning developed by Wenger (1998) for use in the initial training of secondary school mathematics teachers. In this context, I was able to identify the most influential aspects in the learning process of the group of futures teachers: teaching experience and commitment of the participants, their experience in the practicum, the trainers' comments on the transparencies, and the existence and role of the leader.

This study corroborated another of my conjectures: it is possible to study the learning process of the future teachers from a sociocultural perspective. In fact, this perspective enables a characterisation of the development of didactic knowledge that is not possible with traditional schemas. Specifically, this approach determines not only what the group learns but how it learns and on what this learning depends, emphasising the role of context and the interdependent character of learning. The group learned because its members shared commitment to a common goal. To achieve this, they negotiated meanings that were reified in a shared repertoire with which they resolved the tasks assigned.

I thus characterised the development of the didactic knowledge of a group of future teachers from results that could not be obtained in other studies. The results show that, behind the class presentations made by the groups of future teachers who participated in the course and the projects that they handed in to the trainers, there was a complexity inherent in the development of a community of practice. In analysing this complexity systematically and in detail, I identified and characterised many aspects of social learning in the group of future teachers. These characterisations illuminate dimensions in the initial training of secondary school mathematics teachers that often remain opaque in the research literature. They also enable me to explain some of the results from the other studies that form part of this project. For example, they enable me to understand the processes of negotiation of meaning that were reified in the group's transparencies and final project. They also reveal the different positions of the participants and their doubts and confusion, the conflicts that they had to confront and resolve, and the schemas and techniques that they developed to resolve the tasks that they were assigned. Finally, the in-depth analysis of the transcriptions illuminates the group's progress in its commitment to collaborating in the construction of the meanings that they considered necessary to satisfy both the requirements of the course and their interest in becoming mathematics teachers. In this way, I explained and provided evidence to support some of the most important aspects of the development of didactic knowledge of the groups of future teachers that I established in the other studies.

The results of this study show that the group on quadratic function constituted and consolidated a community of practice: in a continuous process of search for and negotiation of meanings, the group established a mutual commitment in the definition of a joint enterprise for which it produced a shared repertoire. The analysis of the transcriptions shows not only that the participants learned and progressed as individuals but also that there was *interde-*

pendent learning: the group, as an entity, progressed in its ability to tackle the tasks at hand, and each participant was concerned about the learning of the others.

Discussion

How can the "proof of existence" (Schoenfeld, 2000, p. 643) that I have just described contribute to the practice of the initial training of secondary school mathematics teachers? I believe that these results can be interpreted and adapted in two areas: evaluation and improvement of the design and development of initial training programmes for secondary school mathematics teachers and reflection on the performance of the trainers of teachers.

Complexity of the Initial Training of Secondary School Mathematics Teachers

One of the clearest conclusions of this research project involves the complexity of the initial training of secondary school mathematics teachers. This complexity is revealed clearly in two issues: the complexity inherent in the notions (tools), concerning whose use the future teachers were expected to be competent, and the complexity of the learning processes that can enable the development of this competence.

The didactic knowledge of the groups of future teachers who participated in the course evolved gradually, heterogeneously, and out of synch with the instruction. The groups of future teachers faced difficulties when they analysed their topic with each of the curriculum organisers of the subject matter analysis. These difficulties were reflected in their productions and performance in a variety of partial meanings that they brought into play in using each of these notions in practice. Some of the groups of future teachers succeeded in overcoming most of the difficulties. However, some of the goals of instruction were not satisfied, in particular with respect to the notion of phenomenology and the practical use of the three curriculum organisers from the subject matter analysis. These difficulties reflected in part the complexity of the process of initial training of secondary school mathematics teachers that took place in the course. The difficulties of the groups of future teachers are the product, among other things, of the complexity of these notions, a complexity demonstrated in the previous sub-section. Nevertheless, the interplay between the technical and practical meanings of the notions also contributes to this complexity.

A group of future teachers transforms a curriculum organiser into an instrument (and thus advances in the development of its didactic knowledge of the notion) to the extent that it negotiates and constructs the meaning of the organiser and develops its technical and practical uses. The process begins with the development of an initial technical use of the notion that is motivated by imitation and supplemented by information from the textbooks. This is the beginning of the process of instrumentalisation (technical). Instrumentation takes place when the technical use is developed, motivated by comments and critiques, in its interaction with the depth of the analysis of the mathematical structure and its putting into practice in other analyses and in the design of the didactic unit (orchestration). This development gives rise to the construction of action plans for the technical analysis of the mathematical structure. Insofar as the capacity to compare and interpret the technical analyses of different mathematical topics is developed, the meaning of the curriculum organiser is constructed. The development of the practical use requires a new process of instrumental genesis. This starts from the information that emerges from the technical analysis of the topic and calls on the orchestration of the different instruments (the curriculum organisers) for the construction of action plans that give rise to the putting into practice of the curriculum organiser to didactic ends.

Trainers as Advisors

My intention in this project was not to evaluate a model for initial training of secondary school mathematics teachers. Thus, I did not seek to respond to questions like, “What works in the classroom?” or “Which method is better?” Rather, I argue that the characterisation of the development of didactic knowledge of the groups of future teachers who participated in the course sheds light on their difficulties and achievements in performing the tasks and on the possible causes of these difficulties and achievements. I believe that this information is relevant both for revision of the design of the course and, subject to corresponding interpretation, for other trainers and other courses that to some extent ground the initial training of secondary school mathematics teachers in a model similar to ours. In the case of our course, the results emphasise two key issues on which it is necessary to improve: the treatment of the phenomenological analysis and the presentation of the practical use of the curriculum organisers and their relation to the technical use of these organisers. On the other hand, they stress the positive role played by the methodological schemas used and by the comments on the future teachers’ transparencies.

The design of the tasks and the comments on the group work can promote a group's interdependent learning *if the group has already been constituted as a community of practice*. However, the members of a group that works with the idea of being a team may interpret the commentaries and the definition of tasks as two additional conditioners of the work routines that they have established, without these factors necessarily promoting negotiation of meaning. If we value the kind of learning that emerges from a community of practice, how do we foster and cultivate this kind of scenario? In the case of our experience, we see that we should change our attitude as trainers. Until now, when we interact with the future teachers (in the classroom or in office hours), our concern has focused on *what* they have learned and on helping them to improve their work (transparencies, presentations and documents). We are now aware that we should take into account the learning processes that give rise to the groups' productions and should develop strategies that promote interdependent learning and negotiation of meaning. We should become "advisors" for the work of the groups. This means that we should be concerned with their learning processes. To achieve this, our attention should not focus only on confirming the extent to which they have developed a shared repertoire and correcting their deficiencies. We should also attend to the factors that can affect both the development of mutual commitment between the members, and the clarity and validity of their joint enterprise. The "Aalborg project model" (Hansen and Jensen, 2004) is one example of this kind of approach to professional training.

The previous proposal leads to a new characterisation of the trainer of teachers. If we tackle the initial training of teachers of secondary school mathematics from the perspective of communities of practice, we should ask ourselves about our competences as trainers. As trainers, we should develop new competences, and this kind of approach imposes new requirements at the institutional level (Beck and Kosnik, 2001, p. 925). What factors affect the "quality" of communities of practice that can be promoted in the initial training of secondary school mathematics teachers? (Llinares and Krainer, 2006, pp. 444-445) What competences should we develop in trainers? What conditions are imposed at the institutional level? These are some of the questions we should tackle in the future.

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